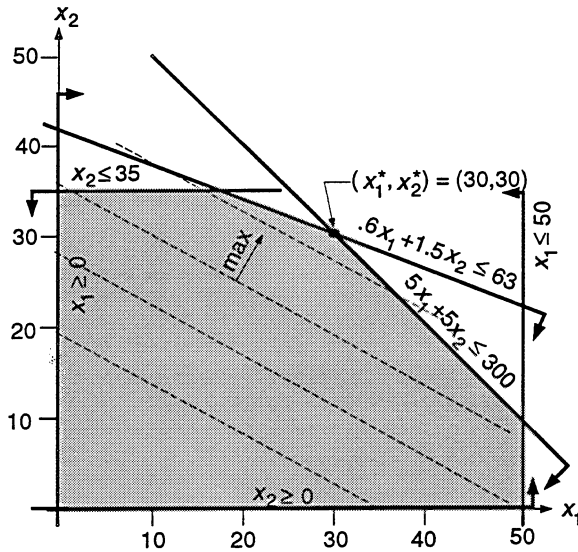


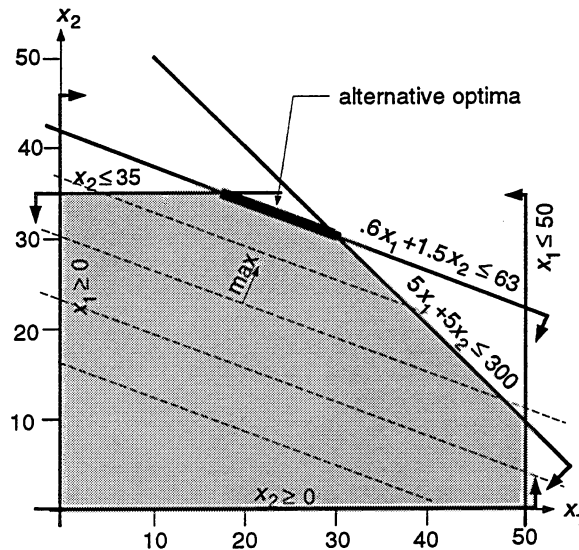
Chapter 2

Exercise Solutions

- 2-1. (a) $\max 200x_1 + 350x_2$ (max total profit),
s.t. $5x_1 + 5x_2 \leq 300$ (legs), $0.6x_1 + 1.5x_2 \leq 63$
(assembly hours), $x_1 \leq 50$ (wood tops), $x_2 \leq 35$
(glass tops), $x_1 \geq 0, x_2 \geq 0$
(b) $x_1^* = \text{basic} = 30, x_2^* = \text{deluxe} = 30$
(c)



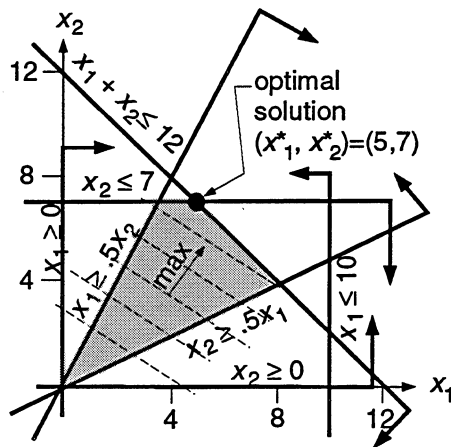
(d)



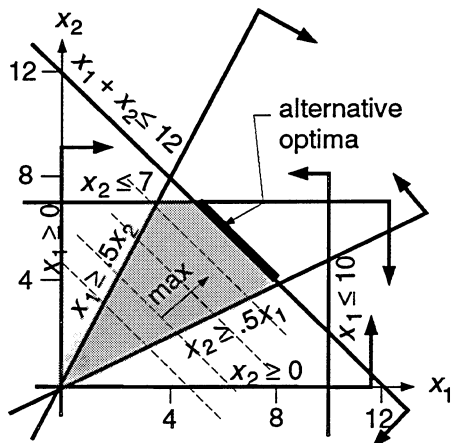
All optimal from $x = (30, 30)$ to $x = (17.5, 35)$.

- 2-2. (a) $\max .11x_1 + .17x_2$ (max total return),
s.t. $x_1 + x_2 \leq 12$ (\$12 million investment),
 $x_1 \leq 10$ (max \$10 million domestic), $x_2 \leq 7$
(max \$7 million foreign), $x_1 \geq .5x_2$ (domestic at
least half foreign), $x_2 \geq .5x_1$ (foreign at least half
domestic), $x_1 \geq 0, x_2 \geq 0$ (b) $x_1^* = \text{domestic} = \5
million, $x_2^* = \text{foreign} = \7 million

(c)



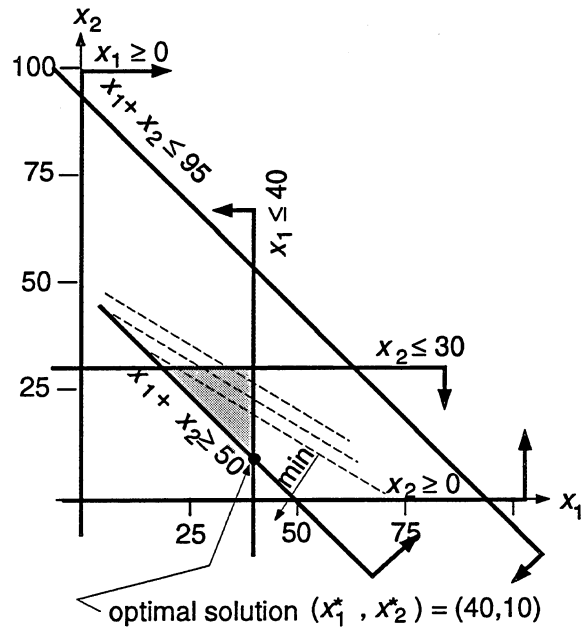
(d)



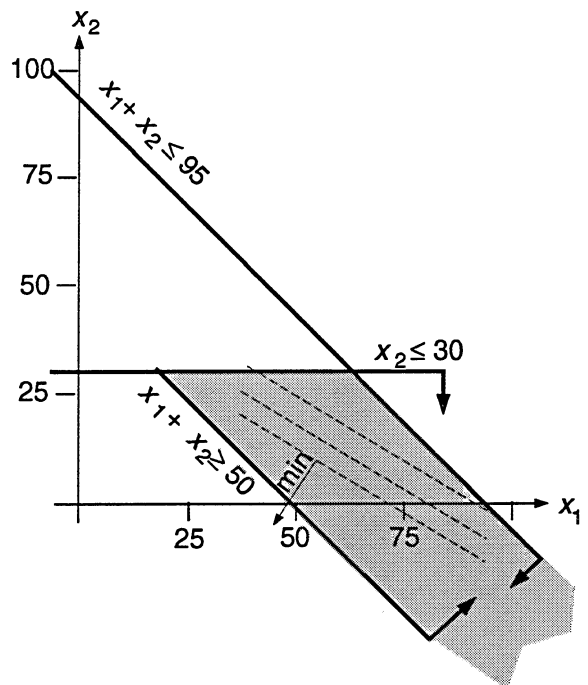
All optimal from $x = (5, 7)$ to $x = (8, 4)$.

2-3. (a) $\min 3x_1 + 5x_2$ (min total cost), s.t. $x_1 + x_2 \geq 50$ (at least 50 thousand acres), $x_1 \leq 40$ (at most 40 thousand from Squawking Eagle), $x_2 \leq 30$ (at most 30 thousand from Crooked Creek), $x_1 \geq 0$, $x_2 \geq 0$ (b) x_1^* =Squawking Eagle=40 thousand, x_2^* =Crooked Creek=10 thousand

(c)

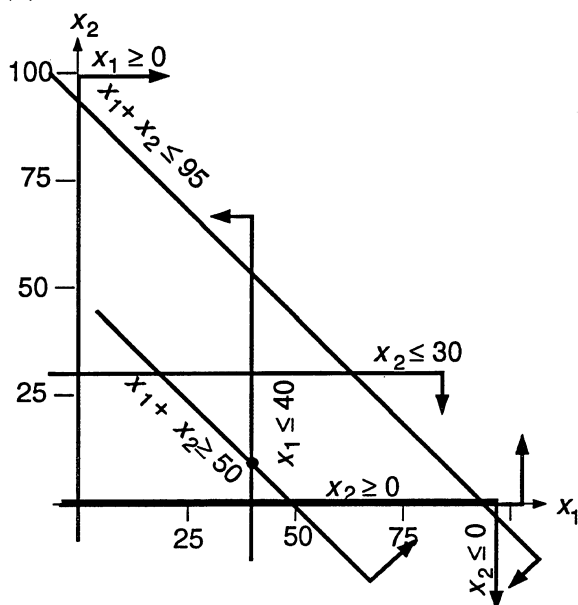


(d)



Improves forever in direction $\Delta x_1 = 1$, $\Delta x_2 = -1$.

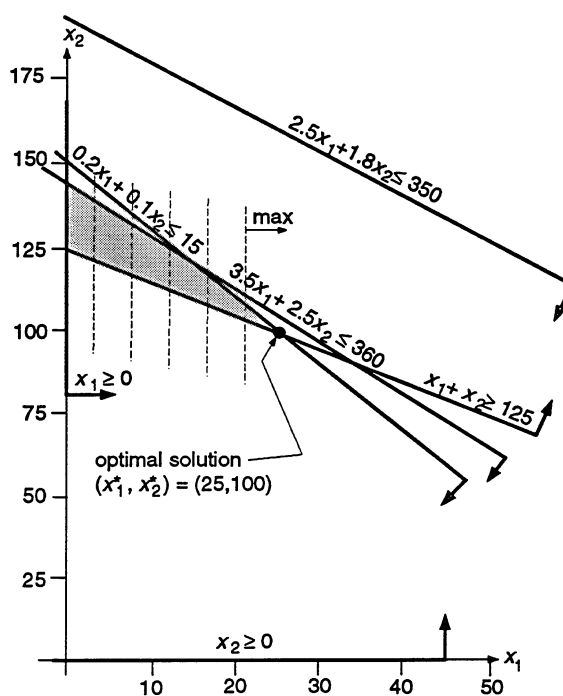
(e)



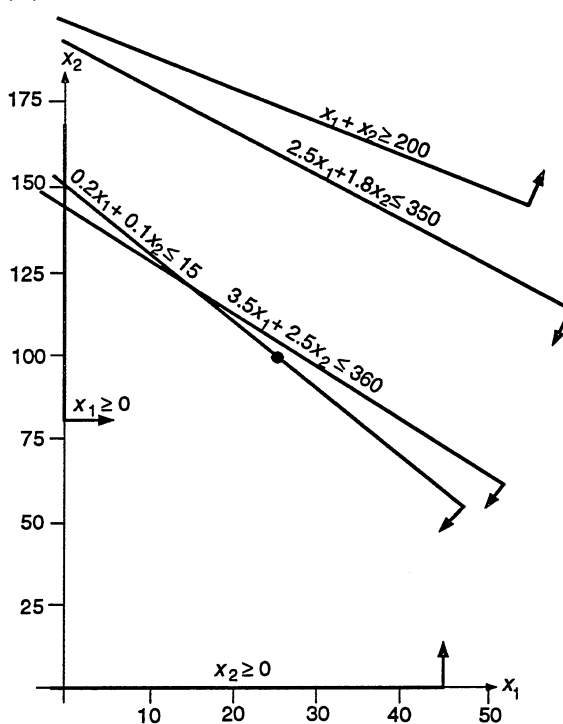
$x_2 = 0$ leaves no feasible.

2-4. (a) max x_1 (max beef content), s.t. $x_1 + x_2 \geq 125$ (weight at least 125), $2.5x_1 + 1.8x_2 \leq 350$ (calories at most 350), $0.2x_1 + 0.1x_2 \leq 15$ (fat at most 15), $3.5x_1 + 2.5x_2 \leq 360$ (sodium at most 360), $x_1 \geq 0$, $x_2 \geq 0$ (b) x_1^* =beef=25g, x_2^* =chicken=100g

(c)

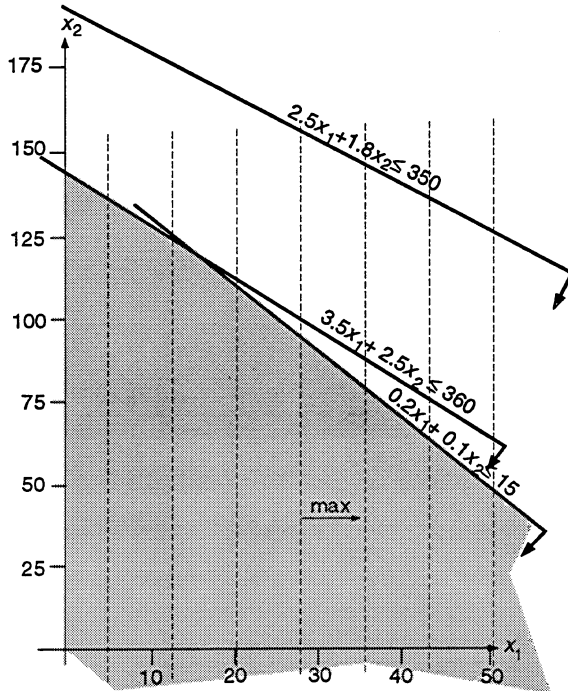


(d)



$x_1 + x_2 \geq 200$ leaves no feasible.

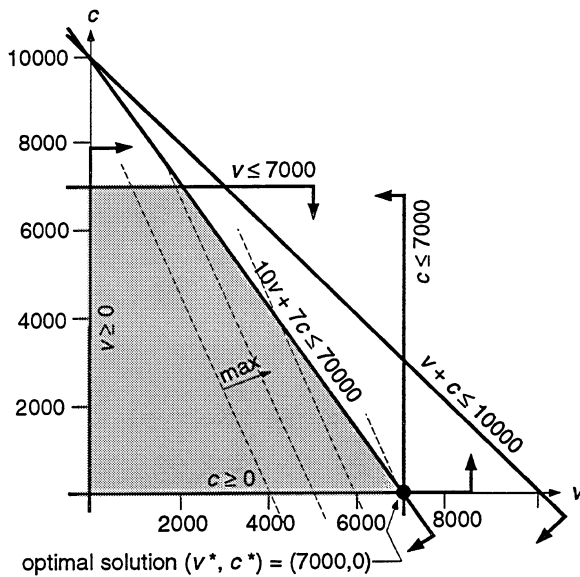
(e)



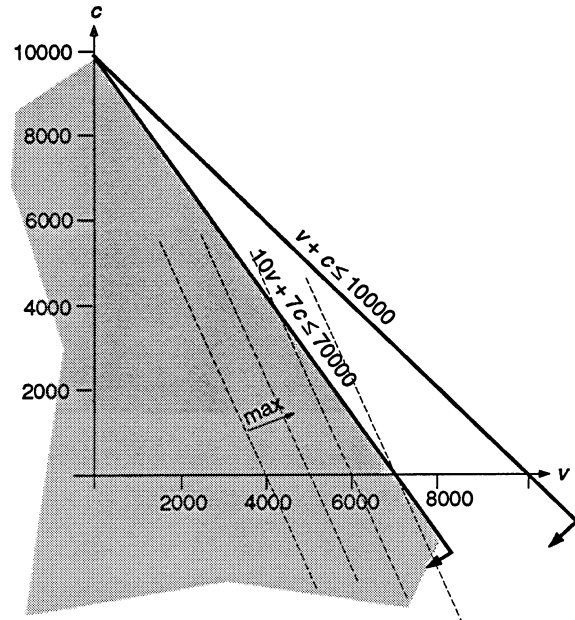
Improve forever in direction $\Delta x_1 = 1$, $\Delta x_2 = -2$.

2-5. (a) max $450v + 200c$ (max total profit), s.t. $10v + 7c \leq 70000$ (water at most 70000 units), $v + c \leq 10000$ (total acreage 10000), $v \leq 7000$ (at most 70% vegetables), $c \leq 7000$ (at most 70% cotton), $v \geq 0$, $c \geq 0$ (b) $v^* = 7000$, $c^* = 0$

(c)

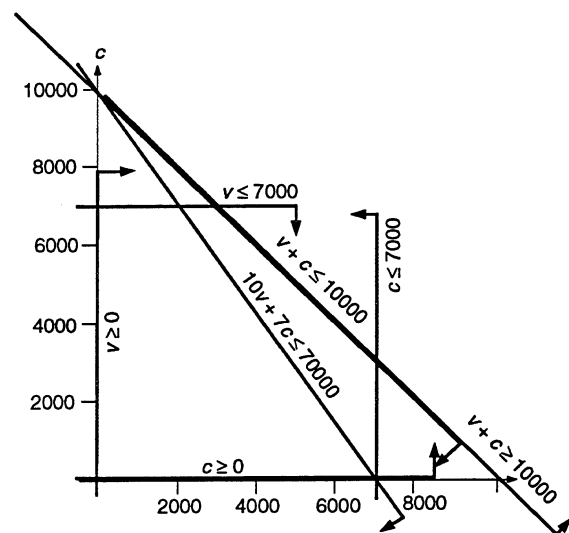


(d)



Improves forever in direction $\Delta v = 10$, $\Delta c = -7$.

(e)



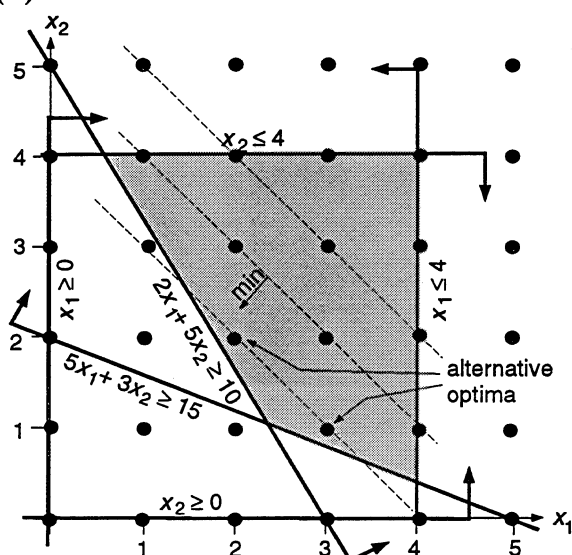
No solution with $v + c = 10000$.

2-6. (a) min $x_1 + x_2$ (min used stock), s.t. $5x_1 + 3x_2 \geq 15$ (cut at least 15 long rolls), $2x_1 + 5x_2 \geq 10$ (cut at least 10 short rolls), $x_1 \leq 4$ (at most 4 times on pattern 1), $x_2 \leq 4$ (at most 4 times on pattern 2), $x_1, x_2 \geq 0$ and integer. (b) Partial cuts make no physical sense because all unused

material is scrap. (c) Either $x_1^* = x_2^* = 2$, or (d)

$x_1^* = 3, x_2^* = 1$

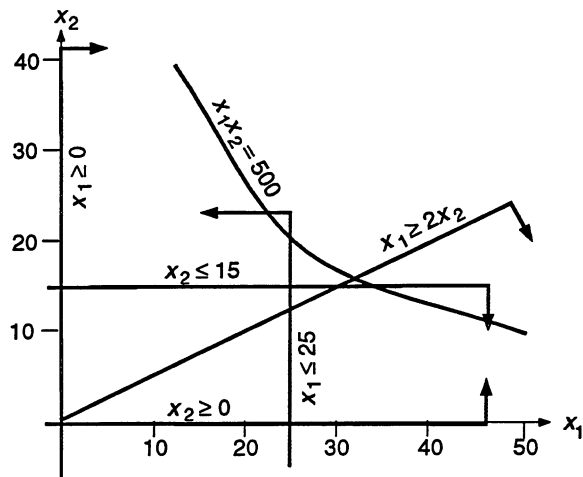
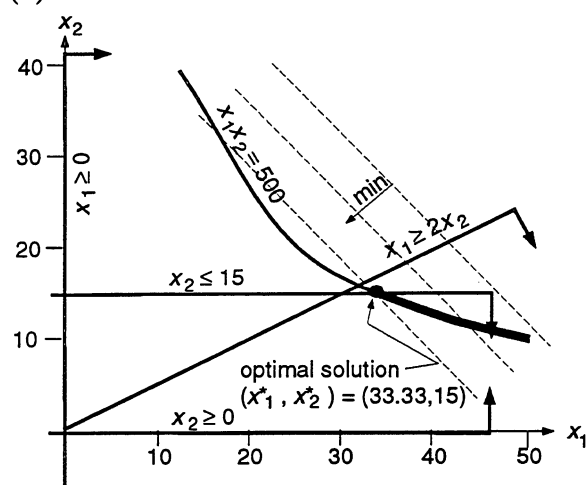
(d)



(e) Both (2, 2) and (3, 1) are feasible and lie on the best contour of the objective.

2-7. (a) min $16x_1 + 16x_2$ (min total wall area), s.t. $x_1x_2 = 500$ (500 sqft pool), $x_1 \geq 2x_2$ (length at least twice width), $x_2 \leq 15$ (width at most 15 ft), $x_1 \geq 0, x_2 \geq 0$ (b) x_1^* =length= $33\frac{1}{3}$ feet, x_2^* =width=15 feet

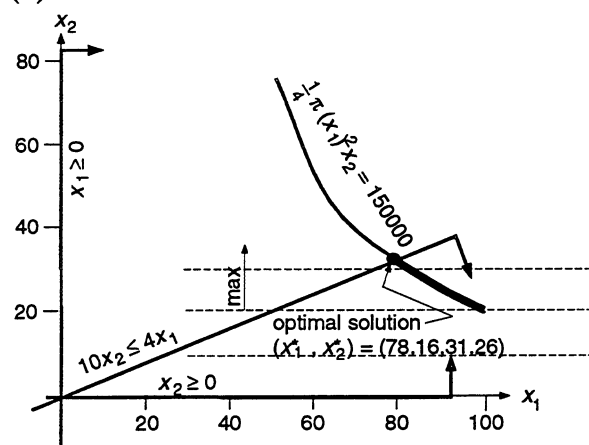
(c)



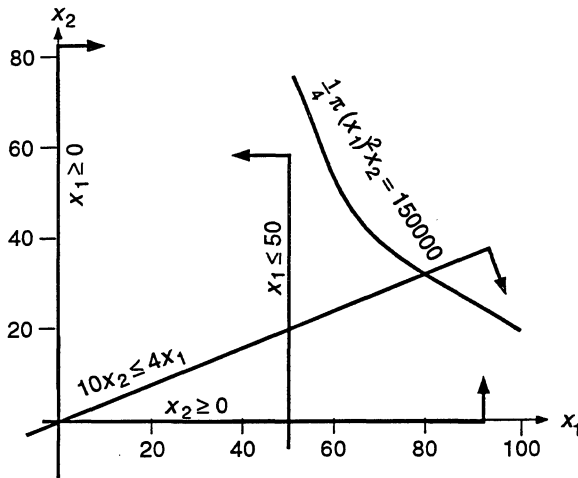
$x_1 \leq 25$ leaves no feasible.

2-8. (a) max x_2 (max number of floors), s.t. $\pi/4(x_1)^2x_2 = 150000$ (150000 sqft floor space), $10x_2 \leq 4x_1$ (height at most 4 times diameter), $x_1 \geq 0, x_2 \geq 0$ (b) x_1^* = diameter = 78.16 feet, x_2^* = floors = 31.26

(c)

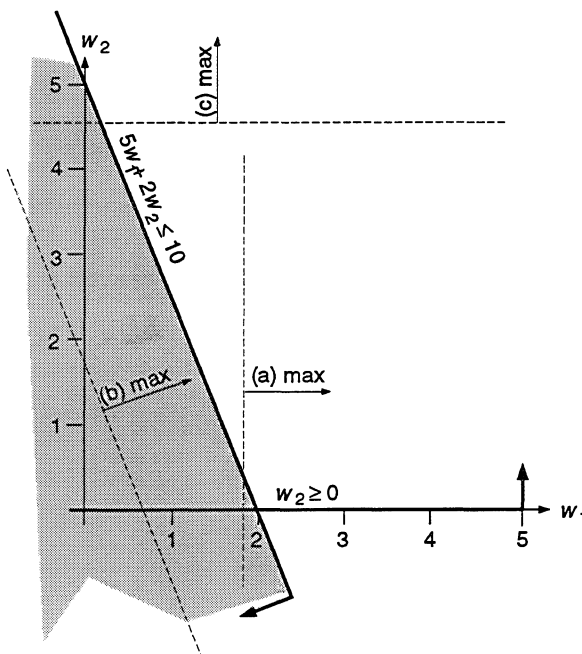


(d)



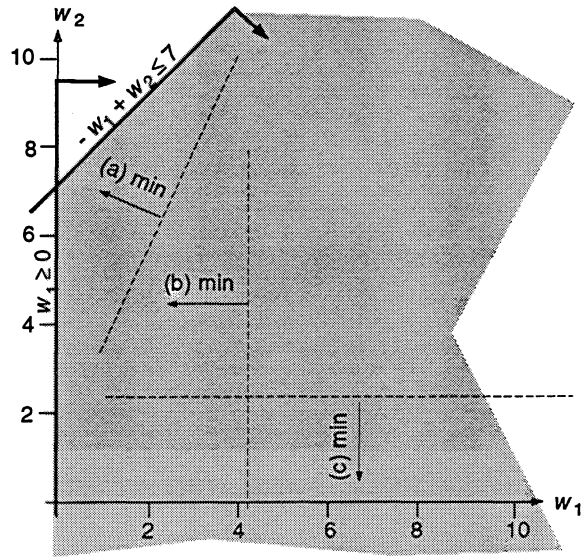
$x_1 \leq 50$ leaves no feasible.

2-9.



(a) max w_1 (b) max $5w_1 + 2w_2$ (c) max w_2

2-10.



(a) min $2w_1 - w_2$ (b) min w_1 (c) min w_2

2-11. (a) min $\sum_{i=3}^4 i \sum_{j=1}^2 y_{i,j}$ (b) max $\sum_{i=1}^4 i y_{3,i}$ (c) max $\sum_{i=1}^p \alpha_i y_{i,4}$ (d) min $\sum_{i=1}^n \beta_i y_i$ (e) $\sum_{j=1}^4 y_{i,j} = s_i, i = 1, \dots, 3$ (f) $\sum_{j=1}^4 a_{i,j} y_j = b_i, i = 1, \dots, 3$ 2-12. (a) $\sum_{i=1}^{17} x_{i,j,t} \leq 200, j = 1, \dots, 5; t = \dots, 7; 35$ constraints(b) $\sum_{j=1}^5 \sum_{t=1}^7 x_{5,j,t} \leq 4000; 1$ constraint(c) $\sum_{j=1}^5 x_{i,j,t} \geq 100, i = 1, \dots, 17; t = 1, \dots, 7; 119$ constraints2-14. (a) $\sum_{j=1}^9 x_{i,j,t} \leq a_i, i = 1, \dots, 47; t = 1, \dots, 10; 470$ constraints(b) $\sum_{i=1}^{47} x_{i,4,t} \geq 1000, t = 1, \dots, 10; 10$ constraints(c) $\sum_{i=1}^{47} \sum_{t=1}^{10} x_{i,2,t} \geq \frac{1}{3} \sum_{i=1}^{47} \sum_{j=1}^9 \sum_{t=1}^{10} x_{i,j,t}; 1$ constraint2-16. (a) $f(y_1, y_2, y_3) \triangleq (y_1)^2 y_2 / y_3, g_1(y_1, y_2, y_3) \triangleq y_1 + y_2 + y_3, b_1 = 13, g_2(y_1, y_2, y_3) \triangleq 2y_1 - y_2 + 9y_3, b_2 = 0, g_3(y_1, y_2, y_3) \triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_3, b_4 = 0$ (b) $f(y_1, y_2, y_3) \triangleq 14y_1 y_2 y_3 + 100, g_1(y_1, y_2, y_3) \triangleq y_1 - 3y_2 + y_3, b_1 = -2, g_2(y_1, y_2, y_3) \triangleq y_1 + 8y_2, b_2 = 10, g_3(y_1, y_2, y_3) \triangleq y_2, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_3, b_4 = 0$

2-17. (a) Linear because LHS is a weighted sum of the decision variables. (b) Linear because

LHS is a weighted sum of the decision variables. (c) Nonlinear because LHS has reciprocal $1/x_9$. (d) Linear because LHS is a weighted sum of the decision variables. (e) Nonlinear because LHS has $(x_j)^2$ terms. (f) Nonlinear because LHS has $\ln(x_3)$ term. (g) Nonlinear because LHS has max operator. (h) Linear because LHS is a weighted sum of the decision variables.

2-18. (a) LP because the objective and all constraints are linear. (b) NLP because of the nonlinear objective function. (c) NLP because of the nonlinear first constraint. (d) LP because the objective and all constraints are linear.

2-19. (a) Continuous because fractions make sense. (b) Discrete because they either win or not. (c) Discrete because a specific process must be used. (d) Continuous because fractions make sense.

2-20. (a) $\sum_{j=1}^8 x_j = 3$ (b) $\sum_{j=1}^5 x_j \geq 2$ (c) $x_3 + x_8 \leq 1$ (d) $x_4 \leq x_1$

2-21. (a) $\max 85x_1 + 70x_2 + 62x_3 + 93x_4$ (max total score), s.t. $700x_1 + 400x_2 + 300x_3 + 600x_4 \leq 1000$ (\$1 million available), $x_j = 0$ or 1 , $j = 1, \dots, 4$ (b) Fund 2 and 4, i.e. $x_1^* = x_3^* = 0$, $x_2^* = x_4^* = 1$

2-22. (a) $\min 200y_1 + 40y_2 + 55y_3 + 75y_4$ (min total land cost), s.t. $y_1 + y_2 \geq 1$ (service NW), $y_1 + y_2 + y_3 \geq 1$ (service SW), $y_1 + y_3 + y_4 \geq 1$ (service capital), $y_1 + y_2 \geq 1$ (service NE), $y_1 + y_4 \geq 1$ (service SE), $y_j = 0$ or 1 , $j = 1, \dots, 4$ (b) Build 2 and 4, i.e. $y_1^* = y_3^* = 0$, $y_2^* = y_4^* = 1$

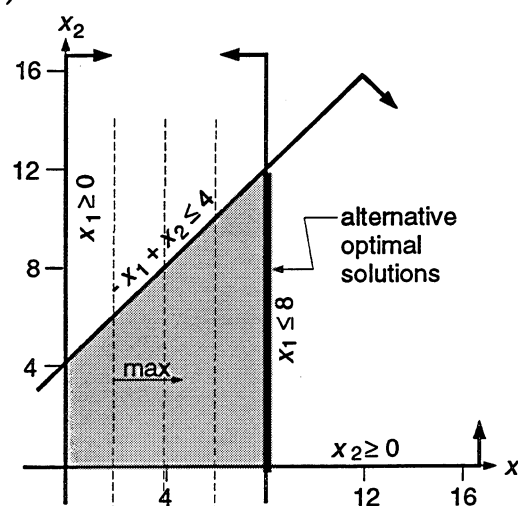
2-23. (a) ILP because the objective and all constraints are linear, but variables are discrete. (b) LP because the objective and all constraints are linear, and all variables are continuous. (c) INLP because the objective is nonlinear and variables are discrete. (d) NLP because the objective is nonlinear and all variables are continuous. (e) INLP because the first constraint is nonlinear and z_3 is discrete. (f) ILP because the objective and all constraints are linear, but variable z_3 is discrete.

2-24. (a) Model (b) because LP's are generally more tractable than ILP's. (b) Model (b)

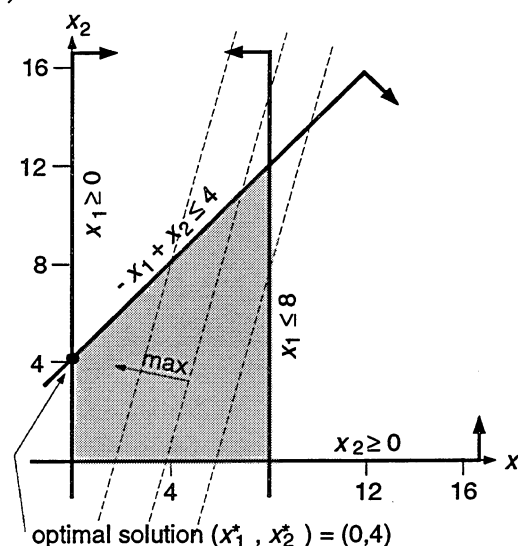
because LP's are generally more tractable than NLP's. (c) Model (d) because NLP's are generally more tractable than INLP's. (d) Model (f) because LP's are generally more tractable than ILP's.

2-25.

(a)



(b)



(c) Helping one can hurt the other.

2-26. (a) $\min .092x_4 + .112x_5 + .141x_6 + .420x_9 + .719x_{12}$ (min total cost), s.t. $x_4 + x_5 + x_6 + x_9 + x_{12} = 16000$ (16000m line), $.279x_4 + .160x_5 + .120x_6 + .065x_9 + .039x_{12} \leq 1600$

(at most 1600 Ohms resistance),
 $.00175x_4 + .00130x_5 + .00161x_6 + .00095x_9 + .00048x_{12} \leq 8.5$ (at most 8.5 dBell attenuation),
 $x_4, x_5, x_6, x_9, x_{12} \geq 0$
 (b) Nonzeros: $x_5^* = 1000, x_{12}^* = 15000$

2-27. (a) Pump rates are the decisions to be made.

(b) $u_j \triangleq$ the capacity of pump j , $c_j \triangleq$ the pumping cost of pump j

(c) $\min \sum_{j=1}^{10} c_j x_j$

(d) $x_1 + x_4 + x_7 \leq 3000$ (well 1), $x_2 + x_5 + x_8 \leq 2500$ (well 2), $x_3 + x_6 + x_9 + x_{10} \leq 7000$ (well 3)

(e) $x_j \leq u_j, j = 1, \dots, 10$

(f) $\sum_{j=1}^{10} x_j \geq 10000$

(g) $x_j \geq 0, j = 1, \dots, 10$

(h) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(i) $x_1^* = x_2^* = x_3^* = 1100, x_4^* = x_6^* = 1500, x_5^* = 1400, x_7^* = 400, x_8^* = x_{10}^* = 0, x_9^* = 1900$

2-28. (a) The decisions to be made are which projects to undertake.

(b) $p_j \triangleq$ the profit for project j , $m_j \triangleq$ the man-days required on project j , and $t_j \triangleq$ the CPU time required on project j .

(c) $\max \sum_{j=1}^8 p_j x_j$

(d) $7 \leq \left(\sum_{j=1}^8 m_j x_j \right) / 240 \leq 10$

(e) $\sum_{j=1}^8 t_j x_j \leq 1000$ (computer time), $\sum_{j=1}^8 x_j \geq 3$ (select at least 3); $x_3 + x_4 + x_5 + x_8 \geq 1$ (include at least 1 of director's favorites)

(f) $x_j = 0$ or $1, j = 1, \dots, 8$

(g) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(h) $x_1^* = x_3^* = x_6^* = x_7^* = 1$, others = 0

2-29. (a) We must decide what quantities to move from surplus sites to fulfill each need.

(b) $s_i \triangleq$ the supply available at i , $r_j \triangleq$ the quantity needed at j , $d_{i,j} \triangleq$ the distance from i to j .

(c) $\min \sum_{i=1}^4 \sum_{j=1}^7 d_{i,j} x_{i,j}$

(d) $\sum_{j=1}^7 x_{i,j} = s_i, i = 1, \dots, 4$

(e) $\sum_{i=1}^4 x_{i,j} = r_j, j = 1, \dots, 7$

(f) $x_{i,j} \geq 0, i = 1, \dots, 4, j = 1, \dots, 7$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzeros: $x_{1,1}^* = 81, x_{1,2}^* = 93, x_{1,3}^* = 166, x_{1,5}^* = 90, x_{1,6}^* = 85, x_{1,7}^* = 145, x_{2,2}^* = 301, x_{3,1}^* = 166, x_{3,4}^* = 105, x_{4,3}^* = 99$

2-30. (a) The values to be chosen are the coefficients in the estimating relationship.

(b) $\min \sum_{j=1}^n \left(c_j - k / (1 + e^{a+bf_j}) \right)^2$ (min total squared error)

(c) Single objective NLP because the objective is quadratic, there are no constraints, and all variables are continuous.

2-31. (a) The decisions to be made are where to assign each teacher.

(b) $\min \sum_{i=1}^{22} \sum_{j=1}^{22} c_{i,j} x_{i,j}$ (min total cost), $\max \sum_{i=1}^{22} \sum_{j=1}^{22} t_{i,j} x_{i,j}$ (max total teacher preference), $\max \sum_{i=1}^{22} \sum_{j=1}^{22} s_{i,j} x_{i,j}$ (max total supervisor preference), $\max \sum_{i=1}^{22} \sum_{j=1}^{22} p_{i,j} x_{i,j}$ (max total principal preference)

(c) $\sum_{j=1}^{22} x_{i,j} = 1, i = 1, \dots, 22$ (each teacher i)

(d) $\sum_{i=1}^{22} x_{i,j} = 1, j = 1, \dots, 22$ (each school j)

(e) $x_{i,j} = 0$ or $1, i, j = 1, \dots, 22$

(f) A multiobjective ILP because the 4 objectives and all constraints are linear, but variables are discrete.

2-32. (a) Each task must go to Assistant 0 or Assistant 1.

(b) $\max 100(1-x_1) + 80x_1 + 85(1-x_2) + 70x_2 + 40(1-x_3) + 90x_3 + 45(1-x_4) + 85x_4 + 70(1-x_5) + 80x_5 + 82(1-x_6) + 65x_6$

(c) $\sum_{j=1}^6 x_j = 3$

(d) $x_5 = x_6$

(e) $x_j = 0$ or $1, j = 1, \dots, 6$

(f) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(g) $x_2^* = x_3^* = x_4^* = 1$, others = 0

2-33. (a) Batch sizes are the decisions to be made.

(b) $\min x_j / d_j, j = 1, \dots, 4$ (each burger j)

(c) $\sum_{j=1}^4 t_j d_j / x_j \leq 60$

(d) $0 \leq x_j \leq u_j, j = 1, \dots, 4$

(e) Multiobjective NLP because the first constraint is nonlinear and all variables are continuous.

2-34. (a) The issue is how many cars to move from where to where.

(b) Relatively large values can be rounded if fractional without much loss, and continuous is more tractable.

(c) $c_{i,j} \triangleq$ the cost of moving a car from i to j , $p_j \triangleq$ the number of cars presently at j , $n_j \triangleq$ the number of cars required at j

(d) $\min \sum_{i=1}^5 \sum_{j=1, j \neq i}^5 c_{i,j} x_{i,j}$

(e) $\sum_{i=1, i \neq k}^5 x_{i,k} - \sum_{j=1, j \neq k}^5 x_{k,j} = n_k - p_k$, $k = 1, \dots, 5$ (each region k)

(f) $x_{i,j} \geq 0$, $i, j = 1, \dots, 5$, $i \neq j$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzero values: $x_{4,2}^* = 115$, $x_{4,3}^* = 165$, $x_{5,1}^* = 85$, $x_{5,3}^* = 225$

2-35. (a) We must decide how much of what fuel to burn at each plant.

(b) $\min \sum_{f=1}^4 \sum_{p=1}^{23} c_{f,p} x_{f,p}$

(c) $\min \sum_{f=1}^4 s_f \sum_{p=1}^{23} x_{f,p}$

(d) $\sum_{f=1}^4 e_f x_{f,p} \geq r_p$, $p = 1, \dots, 23$ (each plant p); 23 constraints

(e) $x_{f,p} \geq 0$, $f = 1, \dots, 4$, $p = 1, \dots, 23$; 92 constraints

(f) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

2-36. (a) The available options are to buy whole logs or green lumber.

(b) Relatively large magnitudes can be rounded without much loss, and continuous is more tractable.

(c) $\min 70x_{10} + 200x_{15} + 620x_{20} + 1.55y_1 + 1.30y_2$

(d) $100(.09)x_{10} + 240(.09)x_{15} + 400(.09)x_{20} + .10y_1 + .08y_2 \geq 2350$

(e) $x_{10} + x_{15} + x_{20} \leq 1500$ (sawing capacity), $100x_{10} + 240x_{15} + 400x_{20} + y_1 + y_2 \leq 26500$ (drying capacity)

(f) $x_{10} \leq 50$ (size 10 log availability), $x_{15} \leq 25$ (size 15 log availability), $x_{20} \leq 10$ (size 20 log availability), $y_1 \leq 5000$ (grade 1 green lumber availability)

(g) $x_{10}, x_{15}, x_{20}, y_1, y_2 \geq 0$

(h) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(i) $x_{10}^* = 50$, $x_{15}^* = 25$, $x_{20}^* = 5$, $y_1^* = 5000$, $y_2^* = 8500$

2-37. (a) Decisions to be made are when to schedule each film.

(b) $\min \sum_{j=1}^{m-1} \sum_{j'=j+1}^m a_{j,j'} \sum_{t=1}^n x_{j,t} x_{j',t}$

(c) $\sum_{t=1}^n x_{j,t} = 1$, $j = 1, \dots, m$ (each film j)

(d) $\sum_{j=1}^m x_{j,t} \leq 4$, $t = 1, \dots, n$ (each time t)

(e) $x_{j,t} = 0$ or 1 , $j = 1, \dots, m$; $t = 1, \dots, n$

(f) A single objective INLP because the one objective is nonlinear, and variables are discrete.

2-38. (a) We need to decide both which offices to open and how to service customers from them.

(b) Offices must either be opened or not.

(c) $f_i \triangleq$ fixed cost of site i , $c_{i,j} \triangleq$ unit cost of audits at j from i , $r_j \triangleq$ required number of audits in state j

(d) $\min \sum_{i=1}^5 \sum_{j=1}^5 c_{i,j} r_j x_{i,j} + \sum_{i=1}^5 f_i y_i$

(e) $\sum_{i=1}^5 x_{i,j} = 1$, $j = 1, \dots, 5$ (each location j)

(f) $x_{i,j} \leq y_i$, $i, j = 1, \dots, 5$ (each site i , location j combination)

(g) $x_{i,j} \geq 0$, $i, j = 1, \dots, 5$, $y_i = 0$ or 1 , $i = 1, \dots, 5$

(h) A single objective ILP because the one objective and all constraints are linear, but the y_i variables are discrete.

(i) Nonzeros: $x_{2,2}^* = x_{2,4}^* = x_{3,1}^* = x_{3,3}^* = x_{5,5}^* = 1$, $y_2^* = y_3^* = y_5^* = 1$

2-39. (a) How to divide funds is the issue.

(b) $\max \sum_{j=1}^n v_j x_j$

(c) $\min \sum_{j=1}^n r_j x_j$

(d) $\sum_{j=1}^n x_j = 1$

(e) $x_j \geq \ell_j$, $j = 1, \dots, n$ (each category j)

(f) $x_j \leq u_j$, $j = 1, \dots, n$ (each category j)

(g) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

2-40. (a) The issue is which module goes to which site.

(b) If $x_{i,j} x_{i',j'} = 1$ the i is at j and i' is at j' , so wire $d_{j,j'}$ will be required. Summing over all possible location pairs captures the wire requirements for i and i' .

(c) \min

$\sum_{i=1}^{m-1} \sum_{i'=i+1}^m a_{i,i'} \sum_{j=1}^n \sum_{j'=1}^n d_{j,j'} x_{i,j} x_{i',j'}$

(d) $\sum_{j=1}^n x_{i,j} = 1$, $i = 1, \dots, m$ (each module i)

(e) $\sum_{i=1}^m x_{i,j} \leq 1$, $j = 1, \dots, n$ (each site j)

(f) $x_{i,j} = 0$ or 1 , $i = 1, \dots, m$, $j = 1, \dots, n$

(g) Single objective INLP because the one objective is nonlinear and variables are discrete.