Chapter 3

3.1 $$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \frac{e\gamma_w}{1+e} = \frac{e\gamma_w}{(1+e)w_{sat}} + n\gamma_w = \boldsymbol{n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w}$$

3.2 $$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \frac{e\gamma_w}{1+e} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$$

Rearranging, $\gamma_{sat}(1+e) = \gamma_d(1+e) + e\gamma_w$

Therefore, $e = \boldsymbol{\dfrac{\gamma_{sat} - \gamma_d}{\gamma_d - \gamma_{sat} + \gamma_w}}$

3.3 $$\gamma_{sat} = \left(\frac{1+w_{sat}}{1+e}\right)G_s\gamma_w = \left(\frac{1+w_{sat}}{1+e}\right)\frac{e\gamma_w}{w_{sat}} = \frac{(1+w_{sat})n\gamma_w}{w_{sat}}$$

Rearranging, $w_{sat}(\gamma_{sat} - n\gamma_w) = n\gamma_w$

Therefore, $w_{sat} = \boldsymbol{\dfrac{n\gamma_w}{\gamma_{sat} - n\gamma_w}}$

3.4 a. $\gamma = \dfrac{W}{V} = \dfrac{12.5}{0.1} = \mathbf{125\ lb/ft^3}$

b. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{125}{1+0.14} = \mathbf{109.64\ lb/ft^3}$

c. $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.71)(62.4)}{109.64} - 1 = \mathbf{0.54}$

d. $n = \dfrac{e}{1+e} = \dfrac{0.54}{1+0.54} = \mathbf{0.35}$

e. $S = \frac{(w)(G_s)}{e} = \frac{(0.14)(2.71)}{0.54} = 0.702 = \mathbf{70.2\%}$

f. Volume of water = $\frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{(125 - 109.64)(0.1)}{62.4} \approx \mathbf{0.024\,ft^3}$

3.5 a. $\gamma = \left(\frac{1+w}{1+e}\right) G_s \gamma_w$; $19.2 = \frac{(1+0.098)(2.69)(9.81)}{1+e}$; e = **0.51**

b. $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+0.51} = \mathbf{17.48\,kN/m^3}$

c. $S = \frac{(w)(G_s)}{e} = \frac{(0.098)(2.69)}{0.51} = 0.517 = \mathbf{51.7\%}$

3.6 a. $\gamma = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(2.69)(9.81) + (0.9)(0.51)(9.81)}{1+0.51} = 20.45\,\text{kN/m}^3$

Water to be added = 20.45 – 19.2 = **1.25 kN/m^3**

b. $\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.69 + 0.51)(9.81)}{1+0.51} = 20.78\,\text{kN/m}^3$

Water to be added = 20.78 – 19.2 = **1.58 kN/m^3**

3.7 a. $V = \frac{\pi}{4}(2.8)^2(22)\left(\frac{1}{12^3}\right) = 0.078\,\text{ft}^3$; $\gamma = \frac{W}{V} = \frac{9.56}{0.078} = \mathbf{122.56\,lb/ft^3}$

b. $w = \frac{W - W_s}{W_s} = \frac{9.56 - 8.51}{8.51} = 0.1233 = \mathbf{12.33\%}$

c. $\gamma_d = \frac{W_s}{V} = \frac{8.51}{0.078} = \mathbf{109.1\,lb/ft^3}$

d. $e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.69)(62.4)}{109.1} - 1 \approx \mathbf{0.54}$

e. $S = \frac{wG_s}{e} = \frac{(0.1233)(2.69)}{0.54} = 0.614 = \mathbf{61.4\%}$

3.8 a. $\gamma = \frac{(1+w)G_s \gamma_w}{1+e} = \frac{(1+w)G_s \gamma_w}{1 + \frac{wG_s}{S}};\ 108 = \frac{(1+0.26)(G_s)(62.4)}{1 + \frac{(0.26)(G_s)}{0.72}};\ G_s = \mathbf{2.72}$

b. $e = \frac{wG_s}{S} = \frac{(0.26)(2.72)}{0.72} = \mathbf{0.98}$

c. $\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.72 + 0.98)(62.4)}{1 + 0.98} = \mathbf{116.6\,lb/ft^3}$

3.9 a. $\gamma_d = \frac{\gamma}{1+w} = \frac{20.6}{1 + 0.166} = \mathbf{17.67\,kN/m^3}$

b. $e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.74)(9.81)}{17.67} - 1 \approx \mathbf{0.52}$

c. $n = \frac{e}{1+e} = \frac{0.52}{1 + 0.52} = \mathbf{0.34}$

d. $S = \frac{wG_s}{e} = \frac{(0.166)(2.74)}{0.52} = 0.874 = \mathbf{87.4\%}$

3.10 a. $\gamma = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(2.74)(9.81) + (0.9)(0.52)(9.81)}{1 + 0.52} = 20.7\,\text{kN/m}^3$

Water to be added = 20.7 – 20.6 = **0.1 kN/m^3**

b. $\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.74 + 0.52)(9.81)}{1 + 0.52} = 21.04\,\text{kN/m}^3$

Water to be added = 21.04 – 20.6 = **0.44 kN/m³**

3.11 a. $\rho_d = \dfrac{\rho}{1+w} = \dfrac{1750}{1+0.23} = \mathbf{1422.76\,kg/m^3}$

b. $e = \dfrac{G_s\rho_w}{\rho_d} - 1 = \dfrac{(2.73)(1000)}{1422.76} - 1 = 0.92$; $n = \dfrac{e}{1+e} = \dfrac{0.92}{1+0.92} = \mathbf{0.48}$

c. $S = \dfrac{wG_s}{e} = \dfrac{(0.23)(2.73)}{0.92} = 0.682 = \mathbf{68.2\%}$

d. $\rho_{\text{sat}} = \dfrac{(G_s+e)\rho_w}{1+e} = \dfrac{(2.73+0.92)(1000)}{1+0.23} \approx 2967\ \text{kg/m}^3$

Water to be added = 2967 – 1750 = **1217 kg/m³**

3.12 a. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{30.75}{0.25(1+0.098)} = \mathbf{112\,lb/ft^3}$

b. $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.66)(62.4)}{112} - 1 \approx \mathbf{0.48}$

c. Volume of water = $\dfrac{(\gamma-\gamma_d)V}{\gamma_w} = \dfrac{\left(\dfrac{30.75}{0.25} - 112\right)(0.25)}{62.4} \approx \mathbf{0.044\ \ ft^3}$

3.13 a. $e = \dfrac{n}{1-n} = \dfrac{0.3}{1-0.3} = \mathbf{0.43}$

b. $\rho_d = \dfrac{G_s\rho_w}{1+e}$; $G_s = \dfrac{\rho_d(1+e)}{\rho_w} = \dfrac{1800(1+0.43)}{1000} = \mathbf{2.57}$

3.14 $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.69)(62.4)}{105} - 1 \approx \mathbf{0.598}$

$$S = \frac{wG_s}{e} = \frac{(0.17)(2.69)}{0.598} = 0.764 = \mathbf{76.4\%}$$

3.15 a. $$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}} = \frac{(1+0.182)(2.67)(62.4)}{1+\frac{(0.182)(2.67)}{0.8}} = \mathbf{122.5\ lb/ft^3}$$

b. $$\gamma_d = \frac{\gamma}{1+w} = \frac{122.5}{1+0.182} = 103.6$$

$$\text{Volume of water} = \frac{(\gamma-\gamma_d)V}{\gamma_w} = \frac{(122.5-103.6)(1)}{62.4} = \mathbf{0.302\ \ ft^3/ft^3\ of\ soil}$$

3.16 a. $$\gamma = \frac{(G_s+Se)\gamma_w}{1+e};\ \ 106 = \frac{(G_s+0.55e)(62.4)}{1+e}$$

$$G_s = 1.148e + 1.698 \qquad \text{(i)}$$

$$114 = \frac{(G_s+0.822e)(62.4)}{1+e} \qquad \text{(ii)}$$

From (i) and (ii): G_s **= 2.73**

b. Using G_s = 2.73 in Equation (i), we get e **= 0.9**

3.17 a. $$D_r = \frac{e_{max}-e}{e_{max}-e_{min}};\ 0.65 = \frac{0.75-e}{0.75-0.52};\ \ e = \mathbf{0.6}$$

b. $$\gamma_d = \frac{G_s\gamma_w}{1+e} = \frac{(2.67)(9.81)}{1+0.6} = \mathbf{16.37\ kN/m^3}$$

3.18 $$D_r = \frac{e_{max}-e}{e_{max}-e_{min}};\ 0.82 = \frac{0.72-e}{0.72-0.46};\ \ e \approx 0.51$$

$$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+0.11)(2.68)(9.81)}{1+0.51} = \mathbf{19.32\ kN/m^3}$$

3.19 $$\gamma_d = \frac{\gamma}{1+w} = \frac{115}{1+0.08} = 106.48\ \text{lb/ft}^3$$

$$D_r = \frac{\left[\frac{1}{\gamma_{d(\min)}}\right] - \left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(\min)}}\right] - \left[\frac{1}{\gamma_{d(\max)}}\right]} = \frac{\left[\frac{1}{92}\right] - \left[\frac{1}{106.48}\right]}{\left[\frac{1}{92}\right] - \left[\frac{1}{108}\right]} = 0.918 = \mathbf{91.8\%}$$

CRITICAL THINKING PROBLEMS

3.C.1 a. $$e = \frac{V_v}{V_s};\quad e_1 + 1 = \frac{V_1}{V_s}$$

$$1+0.92 = \frac{V_1}{V_s};\quad V_1 = 1.92V_s$$

$$1+0.65 = \frac{V_2}{V_s};\quad V_2 = 1.65V_s$$

$$\frac{\Delta V}{V} = \frac{V_1 - V_2}{V_1} = \frac{1.92-1.65}{1.92} = 0.14 = \mathbf{14\%\ (decrease)}$$

b. $$\gamma_{d(1)} = \frac{G_s\gamma_w}{1+e_1} = \frac{G_s\gamma_w}{1+0.92} = \frac{G_s\gamma_w}{1.92}$$

$$\gamma_{d(2)} = \frac{G_s\gamma_w}{1.65}$$

$$\frac{\Delta\gamma_d}{\gamma_{d(1)}} = \frac{\gamma_{d(2)} - \gamma_{d(1)}}{\gamma_{d(1)}} = \frac{\frac{1}{1.65} - \frac{1}{1.92}}{\frac{1}{1.92}} = 0.163 = \mathbf{16.3\%\ \ (increase)}$$

c. $S_1 = \frac{wG_s}{e_1} = \frac{wG_s}{0.92}$; $S_2 = \frac{wG_s}{0.65}$

$$\frac{\Delta S}{S_1} = \frac{S_2 - S_1}{S_1} = \frac{\frac{1}{0.65} - \frac{1}{0.92}}{\frac{1}{0.92}} = 0.415 = \mathbf{41.5\%\ (increase)}$$

3.C.2 a. $D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$

$$e_1 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.47(0.92 - 0.53) = 0.736$$

$$\gamma_d = \frac{G_s\gamma_w}{1+e_1} = \frac{(2.65)(9.81)}{1+0.736} = \mathbf{14.97\,kN/m^3}\ \text{(before compaction)}$$

$$e_2 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.8(0.92 - 0.53) = 0.608$$

$$\gamma_d = \frac{(2.65)(9.81)}{1+0.608} = \mathbf{16.17\ \ kN/m^3}\ \text{(after compaction)}$$

b. $\frac{\Delta H}{H} = \frac{\Delta e}{1+e_1} = \frac{0.736 - 0.608}{1+0.736} = 0.074$; $\Delta H = 0.074H = (0.074)(2) = 0.148\ \text{m}$

Final Height = 2 – 0.148 = **1.852 m**